QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 6945 7044 5888

TIME TAKEN: 82.2955 seconds

OUTPUT:

To solve this problem, we begin by analyzing the given function \( f(x) = | \sin(x) + \tfrac{1}{2}\sqrt{3} | \).

\*\*Step 1: Identify the Points A and B\*\*

We need the coordinates of the first two maxima of \( f \) to the right of the \( y \)-axis. The maxima of \( f(x) \) occur when \( \sin(x) \) is at its maximum value of 1 because adding \( \tfrac{1}{2}\sqrt{3} \) and taking the absolute value will yield the largest possible value of \( f(x) \).

Thus, the maxima occur at \( x = \tfrac{\pi}{2} + 2\pi n \), where \( n \) is an integer. Therefore, the first two maxima to the right of the \( y \)-axis are at:

- \( A = \left( \tfrac{\pi}{2}, f\left( \tfrac{\pi}{2} \right) \right) \)

- \( B = \left( \tfrac{5\pi}{2}, f\left( \tfrac{5\pi}{2} \right) \right) \)

Calculating \( f \) at these points:

\[ f\left( \tfrac{\pi}{2} \right) = \left| 1 + \tfrac{1}{2}\sqrt{3} \right| = 1 + \tfrac{\sqrt{3}}{2} \]

Similarly, \( f\left( \tfrac{5\pi}{2} \right) = 1 + \tfrac{\sqrt{3}}{2} \)

\*\*Step 2: Find \( a \) and \( b \) for \( g(x) = a + b\sin(x) \)\*\*

The function \( g(x) \) reaches its maxima when \( \sin(x) = 1 \), which occurs at the same \( x \)-values as the maxima of \( f(x) \). Therefore, the maxima of \( g(x) \) are at \( x = \tfrac{\pi}{2} + 2\pi n \).

Set up the equations using the condition that \( g \) and \( f \) have the same maxima at these points:

1. At \( x = \tfrac{\pi}{2} \):

\[ a + b \cdot 1 = 1 + \tfrac{\sqrt{3}}{2} \]

So, \( a + b = 1 + \tfrac{\sqrt{3}}{2} \)

2. At \( x = \tfrac{3\pi}{2} \) (where \( \sin(x) = -1 \)):

\[ a + b \cdot (-1) = f\left( \tfrac{3\pi}{2} \right) \]

Calculating \( f \) at \( x = \tfrac{3\pi}{2} \):

\[ f\left( \tfrac{3\pi}{2} \right) = \left| -1 + \tfrac{1}{2}\sqrt{3} \right| = \left| -1 + \tfrac{\sqrt{3}}{2} \right| = 1 - \tfrac{\sqrt{3}}{2} \]

So,

\[ a - b = 1 - \tfrac{\sqrt{3}}{2} \]

\*\*Step 3: Solve for \( a \) and \( b \)\*\*

Adding the two equations:

\[ (a + b) + (a - b) = \left( 1 + \tfrac{\sqrt{3}}{2} \right) + \left( 1 - \tfrac{\sqrt{3}}{2} \right) \]

\[ 2a = 2 \]

\[ a = 1 \]

Subtracting the second equation from the first:

\[ (a + b) - (a - b) = \left( 1 + \tfrac{\sqrt{3}}{2} \right) - \left( 1 - \tfrac{\sqrt{3}}{2} \right) \]

\[ 2b = \sqrt{3} \]

\[ b = \tfrac{\sqrt{3}}{2} \]

Therefore, the exact values are:

\[ a = 1, \quad b = \tfrac{\sqrt{3}}{2} \]

\*\*Answer:\*\* An explicit solution: a = 1 and b = (½)·√3; that is, a = 1 and b = (√3)⁄2

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 9416 9515 8384

TIME TAKEN: 80.4827 seconds

OUTPUT:

To solve this problem, let's go through the given information step by step:

1. \*\*Given function\*\*: \( f(x) = | \sin(x) + \frac{\sqrt{3}}{2} | \).

2. \*\*Points A and B\*\*: These are the points corresponding to the first two maxima of \( f(x) \) to the right of the y-axis.

3. \*\*Function \( g(x) \)\*\*: A sinusoid of the form \( g(x) = a + b \sin(x) \), whose two successive maxima coincide with points A and B.

\*\*Step 1: Find the coordinates of A and B\*\*

First, find where the maxima of \( f(x) \) occur.

Since \( f(x) \) involves the absolute value of \( \sin(x) + \frac{\sqrt{3}}{2} \), the maxima occur where \( \sin(x) + \frac{\sqrt{3}}{2} \) reaches its maximum value.

The maximum value of \( \sin(x) \) is 1, so:

\[

\text{Maximum of } f(x): f\_{\text{max}} = |1 + \frac{\sqrt{3}}{2}| = 1 + \frac{\sqrt{3}}{2}

\]

This occurs at \( x = \frac{\pi}{2} \) (the first maximum to the right of the y-axis).

Since the sine function has a period of \( 2\pi \), the next maximum occurs at \( x = \frac{\pi}{2} + 2\pi \). However, considering the interval \( [0, 2\pi] \), the significant maxima after \( x = \frac{\pi}{2} \) occur at multiples of \( 2\pi \) beyond \( x = \frac{\pi}{2} \).

\*\*Thus, point A is at\*\*:

\[

A = \left( \frac{\pi}{2}, 1 + \frac{\sqrt{3}}{2} \right)

\]

\*\*Point B is at\*\*:

\[

B = \left( \frac{\pi}{2} + 2\pi, 1 + \frac{\sqrt{3}}{2} \right)

\]

\*\*Step 2: Set up equations for \( g(x) \)\*\*

Since \( g(x) \) shares its maxima with \( f(x) \) at points A and B, we have:

1. At \( x = \frac{\pi}{2} \), the maximum of \( g(x) \) is:

\[

g\left( \frac{\pi}{2} \right) = a + b \cdot \sin\left( \frac{\pi}{2} \right) = a + b = 1 + \frac{\sqrt{3}}{2}

\]

2. At \( x = \pi \), which is halfway between A and B, the sine function reaches zero:

\[

g(\pi) = a + b \cdot \sin(\pi) = a + 0 = a

\]

We know that at \( x = \pi \), \( f(\pi) = | \sin(\pi) + \frac{\sqrt{3}}{2} | = \frac{\sqrt{3}}{2} \). Since \( g(\pi) \) should align with \( f(\pi) \) for a smooth sinusoidal fit, we set:

\[

g(\pi) = f(\pi) \implies a = \frac{\sqrt{3}}{2}

\]

\*\*Step 3: Solve for \( a \) and \( b \)\*\*

Using the two equations:

1. \( a + b = 1 + \frac{\sqrt{3}}{2} \)

2. \( a = \frac{\sqrt{3}}{2} \)

Substitute \( a \) from the second equation into the first:

\[

\frac{\sqrt{3}}{2} + b = 1 + \frac{\sqrt{3}}{2}

\]

Subtract \( \frac{\sqrt{3}}{2} \) from both sides:

\[

b = 1

\]

\*\*Final Answer\*\*:

\[

a = \frac{\sqrt{3}}{2} \quad \text{and} \quad b = 1

\]

\*\*Therefore, the values are \( a = \frac{\sqrt{3}}{2} \) and \( b = 1 \).\*\*

Answer:

An explicit solution:  a = (√3) ⁄ 2 and b = 1

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 5512 5611 4480

TIME TAKEN: 54.8264 seconds

OUTPUT:

To solve this problem, we need to find the exact values of \( a \) and \( b \) for the function \( g(x) = a + b\sin(x) \) such that two consecutive maxima of \( g(x) \) coincide with points \( A \) and \( B \), which correspond to the first two maxima of \( f(x) = |\sin(x) + \frac{1}{2}\sqrt{3}| \) to the right of the \( y \)-axis.

\*\*Step 1: Find the Coordinates of Points \( A \) and \( B \)\*\*

First, let's find where the maxima of \( f(x) \) occur.

Since \( f(x) = |\sin(x) + \frac{1}{2}\sqrt{3}| \), the maxima occur when \( \sin(x) + \frac{1}{2}\sqrt{3} \) reaches its maximum positive value. The maximum value of \( \sin(x) \) is \( 1 \), so:

\[

\text{Maximum of } f(x) = \left| 1 + \frac{1}{2}\sqrt{3} \right| = 1 + \frac{1}{2}\sqrt{3}

\]

This occurs at \( x = \frac{\pi}{2} + 2\pi n \). Therefore, the first two maxima to the right of the \( y \)-axis are at:

\[

A = \left( \frac{\pi}{2}, 1 + \frac{1}{2}\sqrt{3} \right)

\]

\[

B = \left( \frac{5\pi}{2}, 1 + \frac{1}{2}\sqrt{3} \right)

\]

\*\*Step 2: Set Up Equations for \( g(x) \)\*\*

Since \( g(x) = a + b\sin(x) \) has maxima where \( \sin(x) = 1 \), the maxima of \( g(x) \) occur at the same \( x \)-values as those of \( f(x) \) (i.e., at \( x = \frac{\pi}{2} \) and \( x = \frac{5\pi}{2} \)).

At these points:

\[

g\left( \frac{\pi}{2} \right) = a + b \cdot 1 = a + b

\]

\[

g\left( \frac{5\pi}{2} \right) = a + b \cdot 1 = a + b

\]

Both maxima of \( g(x) \) must match the maxima of \( f(x) \), so:

\[

a + b = 1 + \frac{1}{2}\sqrt{3} \quad \text{(Equation 1)}

\]

The minima of \( g(x) \) occur where \( \sin(x) = -1 \), at \( x = \frac{3\pi}{2} \). The corresponding value of \( f(x) \) at this point is:

\[

f\left( \frac{3\pi}{2} \right) = \left| -1 + \frac{1}{2}\sqrt{3} \right| = 1 - \frac{1}{2}\sqrt{3}

\]

So the minimum of \( g(x) \) must match this value:

\[

g\left( \frac{3\pi}{2} \right) = a + b \cdot (-1) = a - b = 1 - \frac{1}{2}\sqrt{3} \quad \text{(Equation 2)}

\]

\*\*Step 3: Solve for \( a \) and \( b \)\*\*

Add Equations (1) and (2):

\[

(a + b) + (a - b) = \left( 1 + \frac{1}{2}\sqrt{3} \right) + \left( 1 - \frac{1}{2}\sqrt{3} \right) \\

2a = 2 \\

a = 1

\]

Subtract Equation (2) from Equation (1):

\[

(a + b) - (a - b) = \left( 1 + \frac{1}{2}\sqrt{3} \right) - \left( 1 - \frac{1}{2}\sqrt{3} \right) \\

2b = \sqrt{3} \\

b = \frac{\sqrt{3}}{2}

\]

\*\*Answer:\*\*

The exact values are a = 1 and b = (√3) ⁄ 2.

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 9348 9447 8320

TIME TAKEN: 80.6899 seconds

OUTPUT:

To solve this problem, we need to find exact values of \( a \) and \( b \) in the sine function \( g(x) = a + b\sin(x) \), such that two consecutive maxima of \( g(x) \) coincide with the peaks \( A \) and \( B \) of the function \( f(x) = \left| \sin(x) + \dfrac{1}{2}\sqrt{3} \right| \).

\*\*Step 1: Find the maxima of \( f(x) \)\*\*

First, note that the expression inside the absolute value:

\[

h(x) = \sin(x) + \dfrac{1}{2}\sqrt{3}

\]

The maxima of \( f(x) \) occur when \( h(x) \) is at its maximum positive value. The maximum value of \( \sin(x) \) is 1, so:

\[

\max h(x) = 1 + \dfrac{1}{2}\sqrt{3}

\]

which occurs at \( x = \dfrac{\pi}{2} \).

\*\*Step 2: Determine the points \( A \) and \( B \)\*\*

Since \( f(x) \) is periodic with period \( 2\pi \), the first two maxima to the right of the y-axis occur at:

\[

x\_A = \dfrac{\pi}{2}, \quad x\_B = \dfrac{5\pi}{2}

\]

with their corresponding maxima:

\[

f(x\_A) = f(x\_B) = 1 + \dfrac{1}{2}\sqrt{3}

\]

\*\*Step 3: Find \( a \) and \( b \)\*\*

We know that the maxima of \( g(x) = a + b\sin(x) \) also occur at \( x = \dfrac{\pi}{2} \) and \( x = \dfrac{5\pi}{2} \), and the maximum value of \( g(x) \) is:

\[

g\left(\dfrac{\pi}{2}\right) = a + b \cdot 1 = a + b

\]

Similarly, the minimum value (which is not needed here) would occur at \( x = \dfrac{3\pi}{2} \).

Since the maxima of \( f(x) \) and \( g(x) \) coincide at the same points, we can set them equal:

\[

a + b = 1 + \dfrac{1}{2}\sqrt{3}

\]

We also use the value of \( f \) at \( x = \dfrac{3\pi}{2} \) where \( \sin\left(\dfrac{3\pi}{2}\right) = -1 \):

\[

f\left(\dfrac{3\pi}{2}\right) = \left| -1 + \dfrac{1}{2}\sqrt{3} \right| = \left| -1 + \dfrac{1}{2}\sqrt{3} \right|

\]

Simplify:

\[

f\left(\dfrac{3\pi}{2}\right) = -1 + \dfrac{1}{2}\sqrt{3} \quad (\text{since the expression inside the absolute value is negative})

\]

Now, set \( g\left(\dfrac{3\pi}{2}\right) = f\left(\dfrac{3\pi}{2}\right) \):

\[

a + b(-1) = -1 + \dfrac{1}{2}\sqrt{3}

\]

Simplify:

\[

a - b = -1 + \dfrac{1}{2}\sqrt{3}

\]

\*\*Step 4: Solve for \( a \) and \( b \)\*\*

Now we have two equations:

\[

\begin{cases}

a + b = 1 + \dfrac{1}{2}\sqrt{3} \\

a - b = -1 + \dfrac{1}{2}\sqrt{3}

\end{cases}

\]

Add the two equations:

\[

2a = 0 + \sqrt{3} \implies a = \dfrac{\sqrt{3}}{2}

\]

Subtract the second equation from the first:

\[

2b = 2 \implies b = 1

\]

\*\*Answer:\*\*

An explicit answer: a = 1 and b = (½)·√3; that is, a = 1 and b = (√3)⁄2

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 8996 9095 7936

TIME TAKEN: 71.588 seconds

OUTPUT:

To solve this problem, we need to find the exact values of \(a\) and \(b\) such that the sinusoid \(g(x) = a + b\sin(x)\) has two consecutive maxima coinciding with the points \(A\) and \(B\) of the function \(f(x) = \left| \sin(x) + \frac{\sqrt{3}}{2} \right|\).

\*\*Step 1: Identify the maxima of \(f(x)\)\*\*

First, let's find the points where \(f(x)\) attains its maxima to the right of the y-axis.

Since \(f(x) = \left| \sin(x) + \frac{\sqrt{3}}{2} \right|\), the maxima of \(f(x)\) occur when the expression inside the absolute value is maximized or minimized (since the absolute value will make negative minima positive).

The expression inside the absolute value is maximized when \(\sin(x)\) is maximized:

\[

\sin(x)\_{\text{max}} = 1 \quad \Rightarrow \quad x = \frac{\pi}{2} + 2\pi n, \quad n \in \mathbb{Z}

\]

So the first two maxima of \(f(x)\) to the right of the y-axis occur at:

\[

x\_A = \frac{\pi}{2}, \quad x\_B = \frac{5\pi}{2}

\]

\*\*Step 2: Determine the corresponding values of \(f(x)\)\*\*

Compute \(f(x)\) at these points:

\[

f\left( \frac{\pi}{2} \right) = \left| 1 + \frac{\sqrt{3}}{2} \right| = 1 + \frac{\sqrt{3}}{2}

\]

\[

f\left( \frac{5\pi}{2} \right) = \left| 1 + \frac{\sqrt{3}}{2} \right| = 1 + \frac{\sqrt{3}}{2}

\]

\*\*Step 3: Set up equations for \(g(x)\)\*\*

We require that \(g(x)\) has maxima at the same \(x\)-values and that the maximum values of \(g(x)\) match those of \(f(x)\). The maxima of \(g(x) = a + b\sin(x)\) occur when \(\sin(x) = 1\), so:

\[

g\left( \frac{\pi}{2} \right) = a + b \cdot 1 = a + b

\]

\[

g\left( \frac{5\pi}{2} \right) = a + b

\]

Similarly, the minima occur when \(\sin(x) = -1\):

\[

g\left( \frac{3\pi}{2} \right) = a + b \cdot (-1) = a - b

\]

We know the minimum value of \(f(x)\) at \(x = \frac{3\pi}{2}\):

\[

f\left( \frac{3\pi}{2} \right) = \left| -1 + \frac{\sqrt{3}}{2} \right| = \frac{\sqrt{3}}{2} - 1

\]

\*\*Step 4: Solve for \(a\) and \(b\)\*\*

Set up the equations based on the maxima and minima:

\[

a + b = 1 + \frac{\sqrt{3}}{2} \quad \text{(1)}

\]

\[

a - b = \frac{\sqrt{3}}{2} - 1 \quad \text{(2)}

\]

Add equations (1) and (2):

\[

( a + b ) + ( a - b ) = \left( 1 + \frac{\sqrt{3}}{2} \right) + \left( \frac{\sqrt{3}}{2} - 1 \right)

\]

\[

2a = \sqrt{3}

\]

\[

a = \frac{\sqrt{3}}{2}

\]

Subtract equation (2) from (1):

\[

( a + b ) - ( a - b ) = \left( 1 + \frac{\sqrt{3}}{2} \right) - \left( \frac{\sqrt{3}}{2} - 1 \right)

\]

\[

2b = 2

\]

\[

b = 1

\]

\*\*Answer:\*\*

\[

a = \frac{\sqrt{3}}{2} \quad \text{and} \quad b = 1

\]

An explicit answer: a = (√3)⁄2 and b = 1

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 8095 8194 6976

TIME TAKEN: 66.8231 seconds

OUTPUT:

To solve this problem, we first need to analyze the given function \( f(x) = \left| \sin(x) + \frac{1}{2}\sqrt{3} \right| \) and find the coordinates of the first two maxima, points A and B, to the right of the y-axis.

\*\*Step 1: Identify Points A and B\*\*

The maxima of \( f(x) \) occur when the expression inside the absolute value reaches its maximum positive value. Since \( \sin(x) \) attains its maximum value of 1 at \( x = \frac{\pi}{2} + 2\pi n \), the first maximum A occurs at \( x = \frac{\pi}{2} \). Similarly, because the function is periodic with a period of \( 2\pi \), the next maximum B occurs at \( x = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2} \).

Calculate the corresponding \( y \)-values:

\[

f\left(\frac{\pi}{2}\right) = \left| \sin\left(\frac{\pi}{2}\right) + \frac{1}{2}\sqrt{3} \right| = \left| 1 + \frac{1}{2}\sqrt{3} \right| = 1 + \frac{1}{2}\sqrt{3}

\]

Thus, points A and B are:

\[

A\left( \frac{\pi}{2},\ 1 + \frac{1}{2}\sqrt{3} \right), \quad B\left( \frac{5\pi}{2},\ 1 + \frac{1}{2}\sqrt{3} \right)

\]

\*\*Step 2: Define the Sinusoid \( g(x) \)\*\*

We are given that \( g(x) = a + b \sin(x) \) has two consecutive maxima at points A and B. The maxima of \( g(x) \) occur where \( \sin(x) = 1 \), so:

\[

g\left( \frac{\pi}{2} \right) = a + b \cdot 1 = a + b \\

g\left( \frac{5\pi}{2} \right) = a + b \cdot 1 = a + b

\]

These maxima must match the \( y \)-values of \( f(x) \) at points A and B, so:

\[

a + b = 1 + \frac{1}{2}\sqrt{3} \quad \text{(1)}

\]

\*\*Step 3: Use Additional Information to Solve for \( a \) and \( b \)\*\*

We can find another equation by considering the minimum of \( g(x) \) when \( \sin(x) = -1 \) (occurring at \( x = \frac{3\pi}{2} \)):

\[

g\left( \frac{3\pi}{2} \right) = a + b(-1) = a - b

\]

Similarly, compute \( f\left( \frac{3\pi}{2} \right) \):

\[

f\left( \frac{3\pi}{2} \right) = \left| \sin\left( \frac{3\pi}{2} \right) + \frac{1}{2}\sqrt{3} \right| = \left| -1 + \frac{1}{2}\sqrt{3} \right| = -1 + \frac{1}{2}\sqrt{3}

\]

Note that the absolute value ensures the expression is positive, but since \( \frac{1}{2}\sqrt{3} < 1 \), the result is still negative inside the absolute value, making the positive value \( -1 + \frac{1}{2}\sqrt{3} \).

Set this equal to \( a - b \):

\[

a - b = -1 + \frac{1}{2}\sqrt{3} \quad \text{(2)}

\]

\*\*Step 4: Solve the System of Equations\*\*

Subtract equation (2) from equation (1):

\[

(a + b) - (a - b) = \left( 1 + \frac{1}{2}\sqrt{3} \right) - \left( -1 + \frac{1}{2}\sqrt{3} \right) \\

2b = 2 \\

b = 1

\]

Now substitute \( b = 1 \) into equation (1):

\[

a + 1 = 1 + \frac{1}{2}\sqrt{3} \\

a = \frac{1}{2}\sqrt{3}

\]

\*\*Answer:\*\*

An explicit answer:

  a = (½)·√3 and b = 1; that is, a = ½·√3 and b = 1.

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 6707 6806 5568

TIME TAKEN: 72.5177 seconds

OUTPUT:

To solve this problem, we need to find the exact values of \( a \) and \( b \) for the function \( g(x) = a + b \sin(x) \), such that two consecutive maxima of \( g(x) \) coincide with the points \( A \) and \( B \). These points are the first two maxima of the function \( f(x) = |\sin(x) + \frac{1}{2}\sqrt{3}| \) to the right of the \( y \)-axis.

\*\*Step 1: Identify the maxima of \( f(x) \) and their corresponding \( x \)-values\*\*

1. \*\*First Maxima (\( A \))\*\*:

- At \( x = \frac{\pi}{2} \), \( \sin\left(\frac{\pi}{2}\right) = 1 \).

- Therefore, \( f\left(\frac{\pi}{2}\right) = |1 + \frac{1}{2}\sqrt{3}| = 1 + \frac{1}{2}\sqrt{3} \).

2. \*\*Second Maxima (\( B \))\*\*:

- Since \( f(x) \) has a period of \( 2\pi \) (due to the sine function), the next maxima occur at \( x = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2} \).

- At \( x = \frac{5\pi}{2} \), \( \sin\left(\frac{5\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1 \).

- Therefore, \( f\left(\frac{5\pi}{2}\right) = |1 + \frac{1}{2}\sqrt{3}| = 1 + \frac{1}{2}\sqrt{3} \).

\*\*Step 2: Determine the period of \( g(x) \)\*\*

Since two consecutive maxima of \( g(x) \) coincide with \( A \) and \( B \), the distance between these \( x \)-values is the period of \( g(x) \):

\[ \text{Period} = x\_B - x\_A = \frac{5\pi}{2} - \frac{\pi}{2} = 2\pi \]

This means \( g(x) \) has the standard period of \( 2\pi \).

\*\*Step 3: Set up equations using the coincidences\*\*

1. \*\*At the first maxima \( A \)\*\*:

\[ g\left(\frac{\pi}{2}\right) = a + b \sin\left(\frac{\pi}{2}\right) = a + b = f\left(\frac{\pi}{2}\right) = 1 + \frac{1}{2}\sqrt{3} \]

\[ \Rightarrow a + b = 1 + \frac{1}{2}\sqrt{3} \]

2. \*\*At the minimum between \( A \) and \( B \)\*\* (occurs at \( x = \frac{3\pi}{2} \)):

- \( \sin\left(\frac{3\pi}{2}\right) = -1 \)

- \( f\left(\frac{3\pi}{2}\right) = |-1 + \frac{1}{2}\sqrt{3}| = 1 - \frac{1}{2}\sqrt{3} \)

- \( g\left(\frac{3\pi}{2}\right) = a + b \sin\left(\frac{3\pi}{2}\right) = a - b \)

- Set them equal:

\[ a - b = 1 - \frac{1}{2}\sqrt{3} \]

\*\*Step 4: Solve the system of equations\*\*

Now we have two equations:

1. \( a + b = 1 + \frac{1}{2}\sqrt{3} \)

2. \( a - b = 1 - \frac{1}{2}\sqrt{3} \)

Adding the two equations:

\[ (a + b) + (a - b) = \left(1 + \frac{1}{2}\sqrt{3}\right) + \left(1 - \frac{1}{2}\sqrt{3}\right) \]

\[ 2a = 2 \]

\[ a = 1 \]

Substitute \( a = 1 \) into the first equation:

\[ 1 + b = 1 + \frac{1}{2}\sqrt{3} \]

\[ b = \frac{1}{2}\sqrt{3} \]

\*\*Answer:\*\*

\[ a = 1 \quad \text{and} \quad b = \frac{1}{2}\sqrt{3} \]

An explicit answer:

  a = 1 and b = (½) × √3; that is, a = 1 and b = (1 ⁄ 2)·√3.

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 12631 12730 11712

TIME TAKEN: 132.5731 seconds

OUTPUT:

To solve this problem, we need to find the constants \( a \) and \( b \) for the sinusoidal function \( g(x) = a + b \sin(x) \) such that two consecutive maxima of \( g(x) \) align with the first two maxima of the function \( f(x) = |\sin(x) + \frac{1}{2}\sqrt{3}| \) to the right of the \( y \)-axis.

\*\*Step 1: Identify the maxima of \( f(x) \).\*\*

Since \( f(x) \) includes an absolute value, its maxima occur where \( \sin(x) + \frac{1}{2}\sqrt{3} \) reaches its highest positive values. These are at:

1. \( x = \frac{\pi}{2} \), where \( \sin\left(\frac{\pi}{2}\right) = 1 \):

\[

f\left(\frac{\pi}{2}\right) = \left|1 + \frac{1}{2}\sqrt{3}\right| = 1 + \frac{1}{2}\sqrt{3}

\]

2. \( x = \frac{3\pi}{2} \), where \( \sin\left(\frac{3\pi}{2}\right) = -1 \):

\[

f\left(\frac{3\pi}{2}\right) = \left|-1 + \frac{1}{2}\sqrt{3}\right| = 1 - \frac{1}{2}\sqrt{3}

\]

\*\*Step 2: Set up equations using \( g(x) \).\*\*

The maxima of \( g(x) = a + b \sin(x) \) occur where \( \sin(x) = 1 \) and \( \sin(x) = -1 \). Therefore:

1. At \( x = \frac{\pi}{2} \):

\[

g\left(\frac{\pi}{2}\right) = a + b \cdot \sin\left(\frac{\pi}{2}\right) = a + b = f\left(\frac{\pi}{2}\right)

\]

2. At \( x = \frac{3\pi}{2} \):

\[

g\left(\frac{3\pi}{2}\right) = a + b \cdot \sin\left(\frac{3\pi}{2}\right) = a - b = f\left(\frac{3\pi}{2}\right)

\]

\*\*Step 3: Solve the system of equations.\*\*

Using the values from \( f(x) \):

1. \( a + b = 1 + \frac{1}{2}\sqrt{3} \)

2. \( a - b = 1 - \frac{1}{2}\sqrt{3} \)

Adding the two equations:

\[

( a + b ) + ( a - b ) = \left( 1 + \frac{1}{2}\sqrt{3} \right) + \left( 1 - \frac{1}{2}\sqrt{3} \right)

\]

\[

2a = 2

\]

\[

a = 1

\]

Subtracting the second equation from the first:

\[

( a + b ) - ( a - b ) = \left( 1 + \frac{1}{2}\sqrt{3} \right) - \left( 1 - \frac{1}{2}\sqrt{3} \right)

\]

\[

2b = \sqrt{3}

\]

\[

b = \frac{\sqrt{3}}{2}

\]

\*\*Conclusion:\*\*

The exact values are \( a = 1 \) and \( b = \frac{1}{2}\sqrt{3} \).

\*\*Answer:\*\* An explicit answer: a = 1 and b = (½)·√3; that is, a = 1 and b = (1⁄2)·√3

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 5757 5856 4544

TIME TAKEN: 60.7991 seconds

OUTPUT:

To solve the problem, we need to find the exact values of \( a \) and \( b \) for the sinusoid \( g(x) = a + b\sin(x) \), given that two consecutive maxima of \( g \) coincide with the points \( A \) and \( B \), which are the first two maxima of the function \( f(x) = \left| \sin(x) + \frac{\sqrt{3}}{2} \right| \) to the right of the \( y \)-axis.

\*\*Step 1: Identify the Points \( A \) and \( B \)\*\*

The maxima of \( f(x) \) occur when the expression inside the absolute value is at its maximum. Since \( \sin(x) \) attains its maximum value of 1 at \( x = \frac{\pi}{2} + 2\pi n \), the first two maxima of \( f(x) \) occur at:

- \( x = \frac{\pi}{2} \) (Point \( A \))

- \( x = \frac{5\pi}{2} \) (Point \( B \))

At these points:

\[ f\left(\frac{\pi}{2}\right) = \left| 1 + \frac{\sqrt{3}}{2} \right| = 1 + \frac{\sqrt{3}}{2} \]

\[ f\left(\frac{5\pi}{2}\right) = 1 + \frac{\sqrt{3}}{2} \]

So, both points \( A \) and \( B \) have coordinates:

- \( A = \left( \frac{\pi}{2}, 1 + \frac{\sqrt{3}}{2} \right) \)

- \( B = \left( \frac{5\pi}{2}, 1 + \frac{\sqrt{3}}{2} \right) \)

\*\*Step 2: Set Up Equations for \( a \) and \( b \)\*\*

Since \( g(x) \) has maxima at \( x = \frac{\pi}{2} \) and \( x = \frac{5\pi}{2} \), and these maxima equal \( 1 + \frac{\sqrt{3}}{2} \), we have:

\[ g\left(\frac{\pi}{2}\right) = a + b\sin\left(\frac{\pi}{2}\right) = a + b = 1 + \frac{\sqrt{3}}{2} \]

\[ \Rightarrow a + b = 1 + \frac{\sqrt{3}}{2} \]

The minimum of \( g(x) \) between these maxima occurs at \( x = \frac{3\pi}{2} \), where \( \sin\left(\frac{3\pi}{2}\right) = -1 \). At this point:

\[ g\left(\frac{3\pi}{2}\right) = a + b\sin\left(\frac{3\pi}{2}\right) = a - b \]

We note that \( f(x) \) at \( x = \frac{3\pi}{2} \) is:

\[ f\left(\frac{3\pi}{2}\right) = \left| -1 + \frac{\sqrt{3}}{2} \right| = \left| -\frac{2}{2} + \frac{\sqrt{3}}{2} \right| = \left| -1 + \frac{\sqrt{3}}{2} \right| \]

Computing this:

\[ \left| -1 + \frac{\sqrt{3}}{2} \right| = \left| -1 + 0.866 \right| = \left| -0.134 \right| = 0.134 \]

Assuming \( g \) also shares this minimum value, we have:

\[ a - b = 0.134 \]

\*\*Step 3: Solve for \( a \) and \( b \)\*\*

Now, we have two equations:

1. \( a + b = 1 + \frac{\sqrt{3}}{2} \)

2. \( a - b = 0.134 \)

Adding these equations:

\[ (a + b) + (a - b) = \left( 1 + \frac{\sqrt{3}}{2} \right) + 0.134 \]

\[ 2a = 1 + \frac{\sqrt{3}}{2} + 0.134 \]

Computing the numerical values:

\[ \frac{\sqrt{3}}{2} \approx 0.866 \]

\[ 1 + 0.866 + 0.134 = 2 \]

\[ 2a = 2 \]

\[ a = 1 \]

Substituting back into the first equation:

\[ 1 + b = 1 + \frac{\sqrt{3}}{2} \]

\[ b = \frac{\sqrt{3}}{2} \]

\*\*Answer:\*\*

\[

a = 1 \quad \text{and} \quad b = \frac{\sqrt{3}}{2}

\]

An explicit answer: a = 1 and b = (√3) ⁄ 2; that is, a = 1 and b = (√3)⁄2.

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 5335 5434 4288

TIME TAKEN: 49.9318 seconds

OUTPUT:

To solve this problem, we'll find the values of \( a \) and \( b \) such that the sinusoid \( g(x) = a + b \sin(x) \) has two consecutive maxima at points \( A \) and \( B \), which are the first two maxima of the function \( f(x) = | \sin(x) + \frac{\sqrt{3}}{2} | \) to the right of the \( y \)-axis.

\*\*Step 1: Identify the maxima of \( f(x) \).\*\*

Firstly, notice that \( \frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right) \). The function inside the absolute value of \( f(x) \) can thus be rewritten:

\[

f(x) = | \sin(x) + \sin\left(\frac{\pi}{3}\right) | = 2 \left| \sin\left( \frac{x + \frac{\pi}{3}}{2} \right) \cos\left( \frac{x - \frac{\pi}{3}}{2} \right) \right|

\]

The maxima of \( f(x) \) occur when \( \sin(x) = 1 \), which happens at \( x = \frac{\pi}{2} + 2\pi n \). Therefore, the first two maxima to the right of the \( y \)-axis are at:

\[

x = \frac{\pi}{2} \quad \text{and} \quad x = \frac{5\pi}{2}

\]

At these points, we calculate \( f(x) \):

\[

f\left( \frac{\pi}{2} \right) = f\left( \frac{5\pi}{2} \right) = \left| 1 + \frac{\sqrt{3}}{2} \right| = 1 + \frac{\sqrt{3}}{2}

\]

\*\*Step 2: Relate \( g(x) \) to \( f(x) \).\*\*

Since \( g(x) = a + b \sin(x) \), its maxima occur when \( \sin(x) = 1 \). Therefore, the maximum values of \( g(x) \) at \( x = \frac{\pi}{2} \) and \( x = \frac{5\pi}{2} \) are:

\[

g\left( \frac{\pi}{2} \right) = g\left( \frac{5\pi}{2} \right) = a + b

\]

We set these equal to the maxima of \( f(x) \):

\[

a + b = 1 + \frac{\sqrt{3}}{2} \quad \text{(1)}

\]

Similarly, \( g(x) \) attains its minimum when \( \sin(x) = -1 \), which occurs at \( x = \frac{3\pi}{2} \). At this point:

\[

g\left( \frac{3\pi}{2} \right) = a - b

\]

And from \( f(x) \):

\[

f\left( \frac{3\pi}{2} \right) = \left| -1 + \frac{\sqrt{3}}{2} \right| = 1 - \frac{\sqrt{3}}{2}

\]

Set these equal:

\[

a - b = 1 - \frac{\sqrt{3}}{2} \quad \text{(2)}

\]

\*\*Step 3: Solve for \( a \) and \( b \).\*\*

Add equations (1) and (2):

\[

(a + b) + (a - b) = \left( 1 + \frac{\sqrt{3}}{2} \right) + \left( 1 - \frac{\sqrt{3}}{2} \right)

\]

\[

2a = 2 \implies a = 1

\]

Subtract equation (2) from (1):

\[

(a + b) - (a - b) = \left( 1 + \frac{\sqrt{3}}{2} \right) - \left( 1 - \frac{\sqrt{3}}{2} \right)

\]

\[

2b = \sqrt{3} \implies b = \frac{\sqrt{3}}{2}

\]

\*\*Answer:\*\*

An explicit answer:

 a = 1 and b = (√3) ⁄ 2; that is, a = 1 and b = (√3)⁄2

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 10337 10436 9536

TIME TAKEN: 86.7048 seconds

OUTPUT:

To solve this problem, we need to find constants \( a \) and \( b \) such that the function \( g(x) = a + b\sin(x) \) has two consecutive maxima at the points \( A \) and \( B \), which are the first two maxima of the function \( f(x) = \left| \sin(x) + \frac{1}{2}\sqrt{3} \right| \) to the right of the \( y \)-axis.

1. \*\*Identify the Maxima of \( f(x) \)\*\*:

The maxima of \( f(x) \) occur when the expression inside the absolute value reaches its maximum positive value. Since \( \sin(x) \leq 1 \), the maximum occurs at \( \sin(x) = 1 \).

- At \( x = \frac{\pi}{2} \), \( \sin\left( \frac{\pi}{2} \right) = 1 \), so:

\[ f\left( \frac{\pi}{2} \right) = \left| 1 + \frac{\sqrt{3}}{2} \right| = 1 + \frac{\sqrt{3}}{2} \]

The next maximum occurs at \( x = \frac{\pi}{2} + 2\pi \), since the period of \( f(x) \) is \( 2\pi \).

2. \*\*Set Up Equations Using \( A \) and \( B \)\*\*:

The maxima of \( g(x) \) occur at the same \( x \)-values as those of \( f(x) \) (since they coincide at \( A \) and \( B \)), where \( \sin(x) = 1 \).

Therefore, at these points:

\[ g\left( \frac{\pi}{2} \right) = a + b\sin\left( \frac{\pi}{2} \right) = a + b = 1 + \frac{\sqrt{3}}{2} \]

\[ g\left( \frac{3\pi}{2} \right) = a + b\sin\left( \frac{3\pi}{2} \right) = a - b = 1 - \frac{\sqrt{3}}{2} \]

3. \*\*Solve for \( a \) and \( b \)\*\*:

From the two equations, we can solve for \( a \) and \( b \):

- Adding the equations:

\[ (a + b) + (a - b) = \left( 1 + \frac{\sqrt{3}}{2} \right) + \left( 1 - \frac{\sqrt{3}}{2} \right) \]

\[ 2a = 2 \implies a = 1 \]

- Subtracting the equations:

\[ (a + b) - (a - b) = \left( 1 + \frac{\sqrt{3}}{2} \right) - \left( 1 - \frac{\sqrt{3}}{2} \right) \]

\[ 2b = \sqrt{3} \implies b = \frac{\sqrt{3}}{2} \]

\*\*Answer: Exact values: a = 1 and b = (√3) divided by 2; that is, a = 1 and b = (√3)⁄2\*\*

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 7853 7952 7104

TIME TAKEN: 87.2298 seconds

OUTPUT:

To solve this problem, we first need to analyze the function \( f(x) = \left| \sin(x) + \frac{\sqrt{3}}{2} \right| \) and determine the coordinates of points \( A \) and \( B \), which are the first two maxima of \( f \) to the right of the y-axis.

Given that \(\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\), we can express \( f(x) \) as \( f(x) = \left| \sin(x) + \sin\left(\frac{\pi}{3}\right) \right| \).

Using the identity \( \sin(A) + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \), we get:

\[

f(x) = \left| 2 \sin\left( \frac{x + \frac{\pi}{3}}{2} \right) \cos\left( \frac{x - \frac{\pi}{3}}{2} \right) \right|

\]

This expression shows that \( f(x) \) has a period of \( 2\pi \) and reaches its maximum when \( \sin(x) = 1 \), which occurs at \( x = \frac{\pi}{2} \) and every \( 2\pi \) thereafter.

Thus, the first maximum \( A \) occurs at \( x = \frac{\pi}{2} \), and the second maximum \( B \) occurs at \( x = \frac{\pi}{2} + 2\pi \).

For the sinusoidal function \( g(x) = a + b\sin(x) \) to have maxima at the same points \( A \) and \( B \), it must satisfy:

\[

\begin{cases}

a + b = f\left( \frac{\pi}{2} \right) \\

a - b = f\left( \frac{3\pi}{2} \right)

\end{cases}

\]

Since \( f\left( \frac{\pi}{2} \right) = 1 + \frac{\sqrt{3}}{2} \) and \( f\left( \frac{3\pi}{2} \right) = \left| -1 + \frac{\sqrt{3}}{2} \right| = \frac{2 - \sqrt{3}}{2} \), we have:

\[

\begin{cases}

a + b = 1 + \frac{\sqrt{3}}{2} \\

a - b = \frac{2 - \sqrt{3}}{2}

\end{cases}

\]

Adding the two equations gives \( 2a = 2 \), so \( a = 1 \). Substituting back, we find \( b = \frac{\sqrt{3}}{2} \).

\*\*Answer:\*\* An explicit answer: a = 1 and b = ½ × √3; that is, a = 1 and b = (√3)⁄2

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 10222 10321 9344

TIME TAKEN: 111.2122 seconds

OUTPUT:

To solve this problem, we are given \( f(x) = | \sin(x) + \tfrac{1}{2}\sqrt{3} | \) and \( g(x) = a + b\sin(x) \), and we need to find the exact values of \( a \) and \( b \) such that two successive maxima of \( g(x) \) coincide with points \( A \) and \( B \), which are the first two maxima of \( f(x) \) to the right of the \( y \)-axis.

Firstly, let's find the maxima of \( f(x) \) and their corresponding \( x \)-values. The maxima of \( f(x) \) occur where \( \sin(x) + \tfrac{1}{2}\sqrt{3} \) attains its maximum and minimum values before taking the absolute value.

The maximum of \( \sin(x) \) is \( 1 \), so:

\[

f\left( \frac{\pi}{2} \right) = \left| 1 + \tfrac{1}{2}\sqrt{3} \right| = 1 + \tfrac{1}{2}\sqrt{3}

\]

The minimum of \( \sin(x) \) is \( -1 \), so:

\[

f\left( \frac{3\pi}{2} \right) = \left| -1 + \tfrac{1}{2}\sqrt{3} \right| = 1 - \tfrac{1}{2}\sqrt{3}

\]

These are the first two maxima of \( f(x) \) to the right of the \( y \)-axis at \( x = \frac{\pi}{2} \) and \( x = \frac{3\pi}{2} \).

Now, for \( g(x) \) to have maxima at the same \( x \)-values, its maxima must occur at \( x = \frac{\pi}{2} \) and \( x = \frac{3\pi}{2} \), where the sine function reaches \( 1 \) and \( -1 \), respectively.

At these points:

\[

g\left( \frac{\pi}{2} \right) = a + b \cdot 1 = a + b \\

g\left( \frac{3\pi}{2} \right) = a + b \cdot (-1) = a - b

\]

Since these must match the maxima of \( f(x) \):

\[

a + b = 1 + \tfrac{1}{2}\sqrt{3} \\

a - b = 1 - \tfrac{1}{2}\sqrt{3}

\]

Now, solve these two equations:

\[

\begin{align\*}

a + b &= 1 + \tfrac{1}{2}\sqrt{3} \quad (1) \\

a - b &= 1 - \tfrac{1}{2}\sqrt{3} \quad (2) \\

\end{align\*}

\]

Add equations (1) and (2):

\[

( a + b ) + ( a - b ) = \left( 1 + \tfrac{1}{2}\sqrt{3} \right) + \left( 1 - \tfrac{1}{2}\sqrt{3} \right) \\

2a = 2 \\

a = 1

\]

Now, substitute \( a \) back into equation (1):

\[

1 + b = 1 + \tfrac{1}{2}\sqrt{3} \\

b = \tfrac{1}{2}\sqrt{3}

\]

\*\*Answer:\*\* An explicit solution: a = 1 and b = (½) × √3; that is, a = 1 and b = ½·√3.

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 7556 7655 6400

TIME TAKEN: 60.6457 seconds

OUTPUT:

To solve this problem, we need to find the exact values of \(a\) and \(b\) such that the sinusoid \(g(x) = a + b\sin(x)\) has maxima that coincide with the points \(A\) and \(B\), which are the first two maxima of the function \(f(x) = |\sin(x) + \frac{\sqrt{3}}{2}|\) to the right of the \(y\)-axis.

\*\*Step 1: Identify the Points \(A\) and \(B\)\*\*

First, let's find the \(x\)-coordinates where \(f(x)\) attains its maxima.

Since \(\sin(x)\) reaches its maximum value of \(1\) at \(x = \frac{\pi}{2}\) and every \(2\pi\) thereafter, and given the function \(f(x) = |\sin(x) + \frac{\sqrt{3}}{2}|\), the maxima of \(f(x)\) occur at:

1. \(x = \frac{\pi}{2}\) with \(f\left(\frac{\pi}{2}\right) = \left|1 + \frac{\sqrt{3}}{2}\right| = 1 + \frac{\sqrt{3}}{2}\)

2. \(x = \frac{5\pi}{2}\) with \(f\left(\frac{5\pi}{2}\right) = \left|1 + \frac{\sqrt{3}}{2}\right| = 1 + \frac{\sqrt{3}}{2}\)

Thus, the points are:

- \(A\left(\frac{\pi}{2},\,1 + \frac{\sqrt{3}}{2}\right)\)

- \(B\left(\frac{5\pi}{2},\,1 + \frac{\sqrt{3}}{2}\right)\)

\*\*Step 2: Find \(a\) and \(b\) for \(g(x) = a + b\sin(x)\)\*\*

We know that \(g(x)\) has maxima at the same \(x\)-values as \(f(x)\), so the maxima of \(g(x)\) occur at \(x = \frac{\pi}{2}\) and \(x = \frac{5\pi}{2}\).

At these points:

- \(\sin\left(\frac{\pi}{2}\right) = 1\)

- \(\sin\left(\frac{5\pi}{2}\right) = 1\)

Thus, the maximum values of \(g(x)\) are:

- \(g\left(\frac{\pi}{2}\right) = a + b\)

- \(g\left(\frac{5\pi}{2}\right) = a + b\)

Set these equal to the maximum values of \(f(x)\):

1. \(a + b = 1 + \frac{\sqrt{3}}{2}\)

Next, consider the minimum of \(g(x)\), which occurs when \(\sin(x) = -1\) at \(x = \frac{3\pi}{2}\):

- \(\sin\left(\frac{3\pi}{2}\right) = -1\)

- \(g\left(\frac{3\pi}{2}\right) = a - b\)

Similarly, find \(f\left(\frac{3\pi}{2}\right)\):

- \(f\left(\frac{3\pi}{2}\right) = \left|-1 + \frac{\sqrt{3}}{2}\right| = \frac{2 - \sqrt{3}}{2}\)

Set this equal to \(g\left(\frac{3\pi}{2}\right)\):

2. \(a - b = \frac{2 - \sqrt{3}}{2}\)

\*\*Step 3: Solve for \(a\) and \(b\)\*\*

Now we have two equations:

1. \(a + b = 1 + \frac{\sqrt{3}}{2}\)

2. \(a - b = \frac{2 - \sqrt{3}}{2}\)

Adding the two equations:

\[

( a + b ) + ( a - b ) = \left(1 + \frac{\sqrt{3}}{2}\right) + \left(\frac{2 - \sqrt{3}}{2}\right)

\]

\[

2a = 1 + \frac{\sqrt{3}}{2} + \frac{2 - \sqrt{3}}{2} = 1 + \frac{2}{2} = 2

\]

\[

a = 1

\]

Substitute \(a = 1\) into the first equation:

\[

1 + b = 1 + \frac{\sqrt{3}}{2} \implies b = \frac{\sqrt{3}}{2}

\]

\*\*Answer:\*\*

An explicit answer: a = 1 and b = (√3) ⁄ 2 —that is, a = 1, b = (√3)⁄2

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 5574 5673 4608

TIME TAKEN: 64.7967 seconds

OUTPUT:

To solve this problem, we need to determine the exact values of \( a \) and \( b \) in the function \( g(x) = a + b\sin(x) \), such that two consecutive maxima of \( g(x) \) coincide with the points \( A \) and \( B \), which are the first two maxima of the function \( f(x) = |\sin(x) + \frac{1}{2}\sqrt{3}| \) to the right of the \( y \)-axis.

\*\*Step 1: Identify the Points \( A \) and \( B \)\*\*

First, we analyze \( f(x) \) to find the points where it attains its maxima. Since \( \sin(x) \) has a maximum value of \( 1 \) at \( x = \frac{\pi}{2} + 2\pi n \) (where \( n \) is an integer), we find:

\[

f\left(\frac{\pi}{2}\right) = \left|1 + \frac{1}{2}\sqrt{3}\right| = 1 + \frac{1}{2}\sqrt{3}

\]

Similarly, the next maximum occurs at \( x = \frac{5\pi}{2} \). Thus, the points \( A \) and \( B \) are:

\[

A\left(\frac{\pi}{2}, 1 + \frac{1}{2}\sqrt{3}\right), \quad B\left(\frac{5\pi}{2}, 1 + \frac{1}{2}\sqrt{3}\right)

\]

\*\*Step 2: Analyze the Function \( g(x) \)\*\*

The function \( g(x) = a + b\sin(x) \) has maxima when \( \sin(x) = 1 \). At these points:

\[

g\_{\text{max}} = a + b

\]

Since these maxima coincide with the points \( A \) and \( B \), we have:

\[

a + b = 1 + \frac{1}{2}\sqrt{3} \quad \text{(Equation 1)}

\]

\*\*Step 3: Find Another Equation Using the Minima\*\*

The minima of \( g(x) \) occur when \( \sin(x) = -1 \). At these points:

\[

g\_{\text{min}} = a - b

\]

Similarly, \( f(x) \) attains a value at \( x = \frac{3\pi}{2} \):

\[

f\left(\frac{3\pi}{2}\right) = \left|-1 + \frac{1}{2}\sqrt{3}\right| = 1 - \frac{1}{2}\sqrt{3}

\]

Since the minima of \( g(x) \) correspond to \( f\left(\frac{3\pi}{2}\right) \), we have:

\[

a - b = 1 - \frac{1}{2}\sqrt{3} \quad \text{(Equation 2)}

\]

\*\*Step 4: Solve the System of Equations\*\*

Now we solve the two equations:

\[

\begin{cases}

a + b = 1 + \frac{1}{2}\sqrt{3} \\

a - b = 1 - \frac{1}{2}\sqrt{3}

\end{cases}

\]

Adding the two equations:

\[

( a + b ) + ( a - b ) = (1 + \frac{1}{2}\sqrt{3}) + (1 - \frac{1}{2}\sqrt{3}) \\

2a = 2 \implies a = 1

\]

Subtracting the second equation from the first:

\[

( a + b ) - ( a - b ) = (1 + \frac{1}{2}\sqrt{3}) - (1 - \frac{1}{2}\sqrt{3}) \\

2b = \sqrt{3} \implies b = \frac{\sqrt{3}}{2}

\]

\*\*Answer:\*\*

An explicit answer:

  a = 1 and b = (½)·√3 —that is, a = 1 and b = (1⁄2)√3.

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 4861 4960 4032

TIME TAKEN: 53.4458 seconds

OUTPUT:

To solve this problem, we'll find the exact values of \( a \) and \( b \) for the sinusoid \( g(x) = a + b\sin(x) \), such that its two consecutive maxima coincide with the points \( A \) and \( B \) of the function \( f(x) = \left| \sin(x) + \dfrac{1}{2}\sqrt{3} \right| \).

\*\*Step 1: Identify the maxima of \( f(x) \)\*\*

First, find where \( f(x) \) attains its maxima. The maxima of \( f(x) \) occur where the expression inside the absolute value is positive and \( \sin(x) \) reaches its maximum value.

- The maxima of \( \sin(x) \) occur at \( x = \dfrac{\pi}{2} + 2\pi n \).

- At these points:

\[

f\left( \dfrac{\pi}{2} \right) = \left| 1 + \dfrac{1}{2}\sqrt{3} \right| = 1 + \dfrac{1}{2}\sqrt{3}

\]

- Therefore, the first two maxima of \( f(x) \) to the right of the \( y \)-axis are at \( x = \dfrac{\pi}{2} \) and \( x = \dfrac{5\pi}{2} \).

\*\*Step 2: Determine \( a \) and \( b \) by matching maxima\*\*

Since \( g(x) \) is a sinusoid with maxima at the same \( x \)-values as \( f(x) \), set the maxima of \( g(x) \) equal to those of \( f(x) \):

1. At \( x = \dfrac{\pi}{2} \):

\[

g\left( \dfrac{\pi}{2} \right) = a + b \cdot 1 = a + b = 1 + \dfrac{1}{2}\sqrt{3}

\]

2. At \( x = \dfrac{3\pi}{2} \) (minimum of \( g(x) \) since \( \sin\left( \dfrac{3\pi}{2} \right) = -1 \)):

\[

g\left( \dfrac{3\pi}{2} \right) = a + b \cdot (-1) = a - b = 1 - \dfrac{1}{2}\sqrt{3}

\]

\*\*Step 3: Solve the system of equations\*\*

Subtract the second equation from the first:

\[

\begin{align\*}

(a + b) - (a - b) &= \left(1 + \dfrac{1}{2}\sqrt{3}\right) - \left(1 - \dfrac{1}{2}\sqrt{3}\right) \\

2b &= \sqrt{3} \\

b &= \dfrac{\sqrt{3}}{2}

\end{align\*}

\]

Then, substitute \( b \) back into one of the equations to solve for \( a \):

\[

a + \dfrac{\sqrt{3}}{2} = 1 + \dfrac{1}{2}\sqrt{3} \implies a = 1

\]

\*\*Answer:\*\*

An explicit solution: a = 1 and b = (1 ⁄ 2) × √3; that is, g(x) = 1 + (½)√3·sin(x).

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 8225 8324 7488

TIME TAKEN: 65.6083 seconds

OUTPUT:

To solve this problem, we first need to analyze the function \( f(x) = \left| \sin(x) + \frac{1}{2}\sqrt{3} \right| \).

\*\*Step 1: Identify the Maxima of \( f(x) \)\*\*

The maxima of \( f(x) \) occur when the expression inside the absolute value reaches its maximum. Since \( \sin(x) \) has a maximum value of 1, the maximum of \( f(x) \) occurs at:

\[

\sin(x) = 1 \implies x = \frac{\pi}{2} + 2\pi k, \quad k \in \mathbb{Z}

\]

Therefore, the first two maxima to the right of the y-axis occur at \( x = \frac{\pi}{2} \) and \( x = \frac{5\pi}{2} \). The corresponding maximum values are:

\[

f\left( \frac{\pi}{2} \right) = \left| 1 + \frac{1}{2}\sqrt{3} \right| = \frac{2 + \sqrt{3}}{2}

\]

\[

f\left( \frac{5\pi}{2} \right) = \frac{2 + \sqrt{3}}{2}

\]

So, points A and B are:

\[

A = \left( \frac{\pi}{2}, \frac{2 + \sqrt{3}}{2} \right), \quad B = \left( \frac{5\pi}{2}, \frac{2 + \sqrt{3}}{2} \right)

\]

\*\*Step 2: Determine the Parameters of \( g(x) = a + b \sin(x) \)\*\*

Since \( g(x) \) has maxima at the same points as \( f(x) \), and the maxima values are the same, we set:

\[

g\left( \frac{\pi}{2} \right) = a + b \cdot 1 = \frac{2 + \sqrt{3}}{2}

\]

\[

g\left( \frac{3\pi}{2} \right) = a + b \cdot (-1) = \frac{-2 + \sqrt{3}}{2}

\]

Subtracting these two equations:

\[

\left( a + b \right) - \left( a - b \right) = \frac{2 + \sqrt{3}}{2} - \frac{-2 + \sqrt{3}}{2} \implies 2b = 2 \implies b = 1

\]

Solving for \( a \):

\[

a + 1 = \frac{2 + \sqrt{3}}{2} \implies a = \frac{2 + \sqrt{3}}{2} - 1 = \frac{\sqrt{3}}{2}

\]

\*\*Answer:\*\*

An explicit answer: a = ½ × √3 and b = 1; that is,

 a = (1⁄2)√3 and b = 1.

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 10677 10776 9856

TIME TAKEN: 92.16 seconds

OUTPUT:

To solve this problem, we need to find the exact values of \( a \) and \( b \) in the sine function \( g(x) = a + b\sin(x) \) such that its two successive maxima coincide with the points \( A \) and \( B \), which are the first two maxima of the function \( f(x) = |\sin(x) + \frac{1}{2}\sqrt{3}| \) to the right of the \( y \)-axis.

First, we analyze the function \( f(x) \). The expression inside the absolute value is \( h(x) = \sin(x) + \frac{\sqrt{3}}{2} \). Since \( \sin(x) \) ranges between \( -1 \) and \( 1 \), \( h(x) \) ranges between \( -1 + \frac{\sqrt{3}}{2} \) and \( 1 + \frac{\sqrt{3}}{2} \).

The function \( f(x) \) reaches its maximum values when \( h(x) \) is at its maximum positive value. This occurs when \( \sin(x) \) is at its maximum, which is \( 1 \), occurring at \( x = \frac{\pi}{2} + 2\pi n \) for integers \( n \). Therefore, the first maximum of \( f(x) \) to the right of the \( y \)-axis is at \( x = \frac{\pi}{2} \), and the maximum value is:

\[

f\left(\frac{\pi}{2}\right) = \left| 1 + \frac{\sqrt{3}}{2} \right| = 1 + \frac{\sqrt{3}}{2}

\]

Similarly, the minimum value occurs when \( \sin(x) = -1 \) at \( x = \frac{3\pi}{2} \), giving:

\[

f\left(\frac{3\pi}{2}\right) = \left| -1 + \frac{\sqrt{3}}{2} \right| = 1 - \frac{\sqrt{3}}{2}

\]

Our sine function \( g(x) = a + b\sin(x) \) must have its maxima at the same \( x \)-values as \( f(x) \), with the same maximum and minimum values. This means:

\[

a + b = 1 + \frac{\sqrt{3}}{2} \quad \text{(maximum of } g(x) \text{)} \\

a - b = 1 - \frac{\sqrt{3}}{2} \quad \text{(minimum of } g(x) \text{)}

\]

Subtracting these equations:

\[

(a + b) - (a - b) = \left(1 + \frac{\sqrt{3}}{2}\right) - \left(1 - \frac{\sqrt{3}}{2}\right) \\

2b = \sqrt{3} \\

b = \frac{\sqrt{3}}{2}

\]

Adding the two equations:

\[

(a + b) + (a - b) = \left(1 + \frac{\sqrt{3}}{2}\right) + \left(1 - \frac{\sqrt{3}}{2}\right) \\

2a = 2 \\

a = 1

\]

Therefore, the exact values are \( a = 1 \) and \( b = \frac{\sqrt{3}}{2} \).

\(\boxed{\begin{cases}a=1,\\ b=\dfrac{\sqrt{3}}{2}\end{cases}}\)

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 15351 15450 14016

TIME TAKEN: 202.1222 seconds

OUTPUT:

To solve this problem, we need to find the exact values of \( a \) and \( b \) such that the sinusoid \( g(x) = a + b \sin(x) \) has two consecutive maxima at points \( A \) and \( B \), which correspond to the first two maxima of the function \( f(x) = \left| \sin(x) + \frac{1}{2}\sqrt{3} \right| \) to the right of the \( y \)-axis.

First, observe that \( f(x) = \left| \sin(x) + \frac{1}{2}\sqrt{3} \right| \). The maxima of \( f(x) \) occur where \( \sin(x) \) is at its maximum value. Since \( \sin(x) \) reaches its maximum value of \( 1 \) at \( x = \frac{\pi}{2} \), the first maximum of \( f(x) \) occurs at \( x = \frac{\pi}{2} \):

\[

f\left( \frac{\pi}{2} \right) = \left| \sin\left( \frac{\pi}{2} \right) + \frac{1}{2}\sqrt{3} \right| = \left| 1 + \frac{1}{2}\sqrt{3} \right| = 1 + \frac{1}{2}\sqrt{3}

\]

The second maximum of \( f(x) \) occurs where \( \sin(x) \) is at its minimum, \( -1 \), which is at \( x = \frac{3\pi}{2} \). However, \( f\left( \frac{3\pi}{2} \right) \) is not a maximum. Instead, we consider the next point where \( \sin(x) \) reaches its next maximum at \( x = \frac{5\pi}{2} \), but since this is beyond the first two points to the right of the \( y \)-axis, we need to consider a different approach.

Consider the value of \( \sin(x) \) at \( x = \pi \):

\[

\sin(\pi) = 0 \quad \text{and} \quad f(\pi) = \left| 0 + \frac{1}{2}\sqrt{3} \right| = \frac{1}{2}\sqrt{3}

\]

But this is less than the previous maximum. The next logical point is \( x = \frac{2\pi}{3} \), where:

\[

\sin\left( \frac{2\pi}{3} \right) = \frac{\sqrt{3}}{2} \quad \text{and} \quad f\left( \frac{2\pi}{3} \right) = \left| \frac{\sqrt{3}}{2} + \frac{1}{2}\sqrt{3} \right| = \sqrt{3}

\]

This gives us the second maximum of \( f(x) \) at \( x = \frac{2\pi}{3} \). Therefore, points \( A \) and \( B \) are:

\[

A: \left( \frac{\pi}{2},\, 1 + \frac{1}{2}\sqrt{3} \right), \quad B: \left( \frac{2\pi}{3},\, \sqrt{3} \right)

\]

Now, for \( g(x) = a + b \sin(x) \) to have maxima at these points, the maxima of \( g(x) \) must occur at \( x = \frac{\pi}{2} \) and \( x = \frac{2\pi}{3} \). However, since \( \sin\left( \frac{\pi}{2} \right) = 1 \) and \( \sin\left( \frac{2\pi}{3} \right) = \frac{\sqrt{3}}{2} \), these are not equal.

We can set up two equations based on \( g(x) \) at these points:

\[

\begin{cases}

g\left( \frac{\pi}{2} \right) = a + b \cdot 1 = 1 + \frac{1}{2}\sqrt{3} \\

g\left( \frac{3\pi}{2} \right) = a + b \cdot (-1) = \left| -1 + \frac{1}{2}\sqrt{3} \right| = 1 - \frac{1}{2}\sqrt{3}

\end{cases}

\]

But since \( g\left( \frac{3\pi}{2} \right) \) should not correspond to the maximum at \( B \), there's an inconsistency. The only way the maxima of \( g(x) \) can coincide with points \( A \) and \( B \) is if \( g(x) \) takes its maxima at \( x = \frac{\pi}{2} \) and \( x = \frac{3\pi}{2} \). Using these points, we set up the following equations:

\[

\begin{cases}

a + b = 1 + \frac{1}{2}\sqrt{3} \\

a - b = 1 - \frac{1}{2}\sqrt{3}

\end{cases}

\]

Adding the equations, we get \( 2a = 2 \), so \( a = 1 \). Subtracting the second equation from the first, we get \( 2b = \sqrt{3} \), so \( b = \frac{\sqrt{3}}{2} \).

\*\*Answer: An explicit solution: a = 1 and b = (√3) ⁄ 2; that is, a = 1, b = (√3)/2.

QUESTION NUMBER: 11

MODEL NUMBER: 1

TOKEN USAGE:

99 9108 9207 7936

TIME TAKEN: 73.8165 seconds

OUTPUT:

To solve this problem, we need to find the exact values of \(a\) and \(b\) such that the function \(g(x) = a + b \sin(x)\) has two consecutive maxima at the points \(A\) and \(B\), which correspond to the first two maxima of \(f(x) = \left| \sin(x) + \dfrac{1}{2}\sqrt{3} \right|\) to the right of the \(y\)-axis.

\*\*Step 1: Identify the Points \(A\) and \(B\)\*\*

First, we analyze \(f(x)\) to find the points where it attains its maxima. Since \(f(x)\) involves an absolute value, we consider:

\[ f(x) = \begin{cases} \sin(x) + \dfrac{1}{2}\sqrt{3}, & \text{if } \sin(x) + \dfrac{1}{2}\sqrt{3} \geq 0 \\ -\sin(x) - \dfrac{1}{2}\sqrt{3}, & \text{if } \sin(x) + \dfrac{1}{2}\sqrt{3} < 0 \end{cases} \]

The function \(f(x)\) reaches its maximum when \(\sin(x)\) is at its maximum value, which is \(1\). Therefore, the first maximum occurs at:

\[ x = \dfrac{\pi}{2} \]

So, point \(A\) is at \(\left( \dfrac{\pi}{2}, f\left( \dfrac{\pi}{2} \right) \right) = \left( \dfrac{\pi}{2}, 1 + \dfrac{1}{2}\sqrt{3} \right) \).

The next maximum to the right occurs after a full period of \(2\pi\) for \(f(x)\), so point \(B\) is at:

\[ x = \dfrac{\pi}{2} + 2\pi = \dfrac{5\pi}{2} \]

But since \(B\) should be the next maximum to the right of \(A\), we confirm \(B\) is at \(\left( \dfrac{5\pi}{2}, f\left( \dfrac{5\pi}{2} \right) \right) \).

\*\*Step 2: Relate \(g(x)\) to \(f(x)\) at Points \(A\) and \(B\)\*\*

Since \(g(x) = a + b \sin(x)\), the maxima occur where \(\sin(x) = 1\). Therefore, at the maxima:

\[ g\left( \dfrac{\pi}{2} \right) = a + b \]

\[ g\left( \dfrac{5\pi}{2} \right) = a + b \]

Since both \(A\) and \(B\) have the same \(y\)-coordinate in \(f(x)\) and \(g(x)\):

\[ a + b = 1 + \dfrac{1}{2}\sqrt{3} \]

\*\*Step 3: Consider the Minima to Find Another Equation\*\*

The minima of \(g(x)\) occur where \(\sin(x) = -1\), at \(x = \dfrac{3\pi}{2}, \dfrac{7\pi}{2}, \dots\). At these points:

\[ g\left( \dfrac{3\pi}{2} \right) = a - b \]

In \(f(x)\), at \(x = \dfrac{3\pi}{2}\):

\[ f\left( \dfrac{3\pi}{2} \right) = \left| -1 + \dfrac{1}{2}\sqrt{3} \right| = 1 - \dfrac{1}{2}\sqrt{3} \]

Therefore:

\[ a - b = 1 - \dfrac{1}{2}\sqrt{3} \]

\*\*Step 4: Solve for \(a\) and \(b\)\*\*

Now, we have the system of equations:

1. \( a + b = 1 + \dfrac{1}{2}\sqrt{3} \)

2. \( a - b = 1 - \dfrac{1}{2}\sqrt{3} \)

Adding both equations:

\[ (a + b) + (a - b) = \left( 1 + \dfrac{1}{2}\sqrt{3} \right) + \left( 1 - \dfrac{1}{2}\sqrt{3} \right) \]

\[ 2a = 2 \]

\[ a = 1 \]

Subtracting the second equation from the first:

\[ (a + b) - (a - b) = \left( 1 + \dfrac{1}{2}\sqrt{3} \right) - \left( 1 - \dfrac{1}{2}\sqrt{3} \right) \]

\[ 2b = \sqrt{3} \]

\[ b = \dfrac{\sqrt{3}}{2} \]

\*\*Answer:\*\*

An explicit solution:  a = 1 and b = (√3) ⁄ 2.

>>